B-003-1161004 Seat No. ________

# M. Sc. (Sem. I) Examination

## March – 2021

## CMT - 1004 : Mathematics

*(Theory of Ordinary Differential Equation)*

**Faculty Code : 003**
**Subject Code : 1161004**

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

*Instructions:*

(1) *Attempt any five questions from the following.*
(2) *There are total ten questions.*
(3) *Each question carries equal marks.*

**1) Answer the following:** **14**

1) Define Linear Differential Equation and Linear Homogenous Differential Equation with an example.
2) Prove that for every $n * n$ real matrix $\exp(A + B) = e^A \cdot e^B$ provided $AB = BA$.
3) State and prove change of scale property in Laplace Transform.
4) Find two linearly Independent solutions of $y'' + y = 0$ on R.
5) Show that $u(t) = \begin{pmatrix} cost \\ -sint \end{pmatrix}$ is a solution matrix of the initial value problem$y' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} y,\ y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.
6) Find $L(Sin(ct));\ \forall c \in C$.
7) State Variation of Constant Formula for First Order Differential Equation.

**2) Answer the following:** **14**

1) If $y_1, y_2$ are solutions of $(1 - x^2)y'' - 2xy' + n(n + 1)y = 0$ with initial condition $y_1(0) = 0,\ y_1'(0) = -1,\ y_2(0) = 1$ and $y_2'(0) = 0$ then find $w(y_1, y_2)\left(\frac{1}{2}\right)$.
2) Define Power Series and Bessel's Function.
3) Determine the largest interval of Exisistance of the solution for the I.V.P for the equation:
$$y''' + (t^2 - 1)^{\frac{1}{2}}y = 0\ with\ y(-1) = 1; y'(-1) = 0; y''(-1) = -1$$
4) Prove that $v_1, v_2, \dots \dots . v_n \in K^n$are linearly dependent if and only if $\det[v_1, v_2, \dots \dots . v_n] \neq 0$.
5) Construct the successive approximation $\phi_0,\ \phi_1,\ \phi_2,\ \phi_3$ to a solution of $y' = cosy$, $y'(0) = 0$.
6) Check whether the Legendre's equation $(1 - t^2)y'' - 2ty' + n(n + 1)y = 0$ has a series solution near 0 or not?
7) Define Heavy Side Function and Show that Laplace Transform is linear.

**3) Answer the following:** **14**

1) Prove that the solution of the I.V.P $y'' - 2ty' + 2ny = 0; y'(0) = 0$ and $y(0) = \frac{2(-1)^m(2m)!}{(m)!}$; where $n = 2m; m \geq 0$ is an integer is a Hermite's Polynomial of degree 2m.
2) State and prove variation of constant formula for scalar linear second order non-homogenous differential equation.

**4) Answer the following:** **14**

1) i) Find $L^{-1}\left(\frac{3z+7}{z^2-2z-3}\right)$ and ii) Find $L(Cosct)$.
2) Solve $y'' + y = t$ ; $y = 1$ and $y' = -2$ $at$ $t = 0$ using Laplace Transform.

**5) Answer the following:** **14**

1) State and Prove Gronwall's Inequality.
2) Define Legendre's polynomial and compute the polynomial for 1st, 2nd, 3rd, 4th and 5th degree.

**6) Answer the following:** **14**

1) Prove that if $\emptyset$ is a solution of the I.V.P: $y' = f(t,y); y(t_0) = y_0$ if and only if $\emptyset$ is a solution of the Voltera's equation $y(t) = y_0 \int_{t_0}^{t} f(s, y(s))ds$.
2) Define Convolution. Further show that if $f \in \mathcal{H}$ and $\frac{f(t)}{t} \in \mathcal{H}$ then $L\left(\frac{f(t)}{t}\right)(z) = \int_z^{\infty}(Lf(w))dw$ for which img(w) is bounded and Re(w) $\rightarrow \infty$ .

**7) Answer the following:** **14**

1) Find $exp(tA)$ for $y' = Ay$ on $R$ where A=$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$ on R.
2) i) Classify and locate all the singularities of
$$t^2y'' + ty' + (n^2 - t^2)y = 0; n \neq 0.$$
ii) Prove that $\Gamma(z) = (z-1)\Gamma(z-1); \forall\, z \in \mathbb{C}$ and $\mathrm{Re}(z) > 1$.

**8) Answer the following:** **14**

1) Find Fundamental Matrix of $y' = A(t)y$ on $(-\infty, \infty)$ where $A(t) = \begin{bmatrix} 2 & -3 & 3 \\ 4 & -5 & 3 \\ 4 & -4 & 2 \end{bmatrix}$ for every $t \in (-\infty, \infty)$
2) Find exp (tA) ; $\forall t \in (-\infty, \infty)$ for the above given matrix using its solution matrix.

**9) Answer the following:** **14**

1) i) If $f(t) = t; 0 \leq t \leq 1$ and $f(t+1) = f(t); \forall t \in [0, \infty)$ then find L(f)(z).
   ii) Find $L(e^t sin^2 t)(z)$.
2) Let A be a constant 2 x 2 complex matrix then prove that there exists a constant 2 x 2 non- singular matrix T such that $T^{-1}AT$ has the following forms:

$$\text{a) } \begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix} \text{ and b) } \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

10) **Answer the following:** **14**

1) State and Prove Abel's Formula.
2) Prove that if $a_0(t), a_1(t), a_2(t)$ which are analytic at $t_0$ and $t_0$ is a regular singular point of $a_0(t)y'' + a_1(t)y' + a_2(t)y = 0$ then given equation can be written in the form $(t - t_0)^2 y'' + (t - t_0)\alpha(t)y' + \beta(t)y = 0$ for some functions $\alpha(t)$ and $\beta(t)$ which are analytic at $t_0$ and not all $\alpha(t_0), \beta(t_0)$ and $\beta'(t_0)$ are zero.

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